WALRASIAN AND NON-WALRASIAN MICROECONOMICS

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Keywords: walrasian, non-walrasian, microeconomics, transactions, effective demand, excess demand.

Contents

Summary

This article addresses the question of how economic agents such as consumers and firms formulate their plans on the basis of prices. The concern is with economic agents operating under perfect competition, i.e. with agents who are unable to influence market prices. The compatibility of such plans is considered, as is the nature of mutually compatible plans. This is the Walrasian part of the discussion. Non-Walrasian methods are also considered, and the transactions that agents make on the basis of prices and quantity constraints are discussed. The question of whether quantity constraints (rationing schemes) may be devised, for any configuration of fixed prices, so that the constrained plans of agents are compatible is discussed following this.

The approach adopted is used to analyze the transactions that take place when agents respond not only to prices, but also to quantity constraints. This method is applied to two areas where Walrasian methods have been relatively unsuccessful. The first is the question of how the market computes the Walrasian configuration. The second is the analysis of unemployment.

1. Introduction

1.2 Decision Makers, Economic Agents and Markets

This article examines the behavior of certain kinds of decision-makers and how they plan out their activities under various scenarios. The decision-makers to be considered are usually called economic agents, and are distinguished on the basis of their activities. An activity that uses goods and/or services (factors) to produce other goods or services is called a production activity. A decision-maker who controls such activities is called a firm. An activity engaged in by agents to meet their own needs is usually called a consumption activity; agents who engage in such activities are referred to as consumers or individuals. There may be other agents who need to decide on the levels of taxation or the amount of money to be spent on national security; such agents are usually identified with the government of the country.

An economy is made up of these different types of economic agents. The economic agents, as should be clear from the above description, generally have different objectives; for instance, firms are usually taken to be interested in the maximization of profits; consumers (or households or individuals) are interested in maximizing their own well being, or utility; the government has its own typical concerns. Because these agents have such very different objectives, the task of coordinating their activities is difficult. In this article, we assume agents operate in markets; that is, agents turn to the markets to make any transactions they might wish. Further, we assume the markets are competitive; i.e. there is no agent in any market who can control the price to his or her own advantage. (See chapter *Strategic Behavior*)

1.2 Walrasian and Non-Walrasian Approaches

Two related but distinct approaches to the analysis of the issues outlined above will be discussed. In the Walrasian Approach, the different agents take only prices as signals, and decide on their plans on the basis of these. As we shall see, if the signals are appropriate the plans that the different agents make will be mutually compatible. The properties of such configurations will form a major part of the discussion below. Walrasian methods consider only that configuration where plans of agents are compatible and the only transactions that are made are those that match.

In the course of the discussion it will become clear that the configuration in which all plans are mutually compatible may be difficult to reach; this raises the question of what happens when the plans made by agents are not compatible, or, to put it differently, when market clearing is not achieved. One approach to this issue has been to introduce various quantity constraints, at some given configuration of prices, which the agents are asked to respond to. If these constraints are appropriate, the constrained plans should be mutually compatible. Non-Walrasian approaches consider the kinds of rationing or quantity constraints that will make the constrained plans mutually compatible. The second part of this article will consider the nature of these quantity constraints and the property of transactions that would be implied by the constraints.

In addition to the one mentioned above, a Non-Walrasian approach has to be considered for another reason. In the Walrasian approach, as pointed out above, all plans made by agents are found to be compatible. All persons who wish to work, for example, will find jobs. There is thus no scope to analyze unemployment. To analyze any problem in economics that involves an observed under-utilization of some available resource, it is essential to step outside the Walrasian paradigm. An example of such an approach to unemployment will be discussed below.

Another problem to be addressed is the question of how competitive markets reach a configuration where the plans are mutually compatible. A view is adopted that competitive agents realize that the plans of agents are not compatible only when they try to transact and are unable to do so. Based on this observation, a process is described whereby a Walrasian configuration may be ultimately achieved.

1.3 Overview

As outlined above, in this article we confine ourselves to the framework of microeconomics, and both Walrasian and Non-Walrasian methods will be discussed. So far as the former is concerned, the notion of demand and supply, and hence, excess demand will be introduced. We begin from the origins of demand and supply, showing how plans are made by agents such as consumers and firms. The desired transactions by these agents, the notion of excess demand, will be taken up, next. In each case, the properties of these constructs will be discussed and the usual assumptions made in such contexts will also be provided. The main point of emphasis is how these diverse plans are mutually compatible.

For the Non-Walrasian Approach, on the other hand, it is the notion of effective demand that is the main construct. There is, however, a misuse of terms here since the term effective demand is not really a different kind of demand; it should perhaps be termed effective excess demand, since the effective demand is actually a kind of constrained transaction that an agent may wish to make. The basic point of departure from the Walrasian approach is that if markets do not clear, i.e. the plans are not mutually compatible, is there any way of devising constraints on transactions so that once these constraints are taken in to account, the constrained transactions match? Naturally the constraints must be meaningful and the article will consider how best to introduce such considerations.

Finally, as discussed above, two applications of the above approach will be considered. One of them is in the context of price formation--a process, whereby agents themselves bid the prices up or down when they fail to make the desired transaction, will be considered. The other application is in the area of unemployment equilibria; the constructs of effective demand for labor and output are analyzed within the context of an aggregative model to show the properties of the resulting unemployment equilibria.

Because of constraints on space, this cannot be an exhaustive treatment. We provide here only a statement of the major results and the assumptions under which they hold. What have been left out are primarily the demonstrations and proofs. Readers wishing to explore these issues will find suitable references in the bibliography.

2. Walrasian Transactions: Excess Demand

2.1 Demand

2.1.1 Individual Demand Functions and their Properties

Demand for goods and services originates from the plans made by individuals when they know their own incomes and the prices of the items they wish to buy and sell. This section considers the formation of such plans and how they change when prices change.

Suppose there are n goods and/or services; an individual decision maker h is assumed to possess a set of all possible consumption possibilities, $X \subseteq \Re^n$, the n-dimensional Euclidean space. It is usual to assume that

Axiom 1. $X \subseteq \Re$ is non-empty, closed, convex, and bounded from below

The individual has some initial resources $e \neq 0$, $e \in X$, the endowment; since the individual must be able to compare various possibilities in X we shall assume that

Axiom 2. X is completely ordered by a binary relation R

Thus the binary relation R is assumed to be reflexive (i.e. for any $x \in A$, xRx), complete (i.e. for any x, $y \in A$, either xRy *or* yRx), transitive (i.e. for any x, y, $z \in A$, $xRy \;\&\; yRz \Rightarrow xRz$). R is usually interpreted to be the "no worse than" relation. Two other binary relations are often derived from it. They are defined as follows. Strict preference P: *if* xRy and $\sim yRx$ then xPy. Indifference *I: if* $xRy \;\&\; yRx$ *then* xIy. "$"\sim yRx"$ is to be read as: it is *not* the case that yRx.

The following axiom, continuity, is also usually imposed on the binary relation R:

Axiom 3. $R_x = \{y \in X : yRx\}$, $R^x = \{y \in X : xRy\}$ are closed subsets of X for every $x \in X$.

Under the above axioms, it can be shown that:

There is a continuous real valued function $U : X \to \Re$ such that $U(x) \geq U(y) \Leftrightarrow xRy$

The function $U(\cdot)$ is called a utility function. The individual's choice over the set X is constrained by his purchasing power, which is measured by $M = p \cdot e$, where $p \in \Re^n_{++}$ denote the market prices. The set of affordable consumption possibilities is provided by the budget set: $B(p, M) = \{y \in X : p \cdot y \leq M\}$; alternatively, the budget set may be denoted by $B(p, e)$. The individual's choice problem may now be characterized by the following problem:

$$\text{Max } U(x)$$

subject to $x \in B(p, M)$

This problem is referred to as the Maximum Utility Problem (MUP); the solution to MUP provides us with the demand at p, M or at p, e; note that $p \in \Re^n_{++} \Rightarrow B(p, M)$ is a compact subset of X; consequently, a continuous function will always attain its bounds in such a set, and hence there will always be a solution to MUP. In general, such a solution need not be unique. Consider, however,

Axiom 4. $x, y \in X$, $x \neq y$, $xRy \Rightarrow (\lambda x + (1-\lambda) y) P\, y \,\forall \lambda,\ 0 < \lambda < 1$

If R satisfies the above, the resulting $U(\cdot)$ is said to be strictly quasi-concave; it is easy to check that with this restriction, the solution to MUP is unique; this unique solution is represented by $f(p, M)$ or by a slight abuse of notation, by $f(p, e)$; this is the *demand function*. The binary relation R is said to satisfy *local non-satiation* if:

Axiom 5. *For* $x \in X$ and for any neighborhood $N(x)$ *of* x, $\exists y \in N(x) \cap X$, $y\, P\, x$

This ensures that in solving MUP, the decision maker must spend all or that $p \cdot f(p, e) = p \cdot e$; note also that $f(\lambda p, e) = f(p, e)$ for any $\lambda > 0$: homogeneity of degree zero in the prices; this follows since multiplying all the prices by some constant does not alter the Budget set $B(p, M)$ (recall, $M = p \cdot e$). Consider next, the continuity of the function $f(p, e)$. A cheaper point exists at (p, e) if there is $\tilde{x} \in X$ such that $p.\tilde{x} < p.e$. Given the existence of a cheaper point at (p, e), one may show that the budget map $B : \Re^n_{++} \times X \to X$ is lower hemi-continuous at (p, e); i.e. for any $z \in B(p, e)$, $\exists$ a sequence $\{p^s, e^s\}$ such that $\{p^s, e^s\} \to (p, e)$ and there is a sequence $\{z^s\}$ such that $z^s \in B(p^s, e^s)$ $\forall s$ and $z^s \to z$. This property is crucial for the demonstration of the fact that $f(p, e)$ is a continuous function of (p, e) whenever there is a cheaper point at (p, e).

The demand function may be seen to have some further properties. These are best demonstrated through the consideration of the following minimization problem:

For a given $p \in \Re^n_{++}, x \in X$

Minimize $p.y$

subject to $y \in R_x$

This problem is the Minimum Expenditure Problem (MEP); since the constraint set is closed (by Axiom 3) and bounded below (by Axiom 1), the MEP has a solution; let the minimum value attained be denoted by $E(p, x)$ or by $E(p, U)$ where $U = U(x)$. The function $E(p, x)$ called the expenditure function determines the minimum expenditure required at prices p to attain the same level of utility as at x. The following property is crucial:

The function $E(p, x)$ is a non-decreasing, concave function of p for any $x \in X$; moreover, $E(\lambda p, x) = \lambda E(p, x)$ for any $\lambda > 0$.

One may now define $B(p, E(p, x))$ and $f(p, E(p, x))$ exactly as above, replacing M by $E(p, x)$; it is to be noted that $U(f(p, E(p, x))) = U(x)$; thus $f(p, E(p, x))$ is the compensated demand function. Much of modern theory of the consumer is based on the relationship between the problems MUP and MEP, often referred to as the "duality" theory. It may be shown, under the assumptions employed, if $\hat{x}$ solves MUP for a given p and M, then the problem MEP with the same p and $R\hat{x}$ in the constraint will be solved by $\hat{x}$; conversely, if given p and some $x \in X$, MEP is solved by some y and $E(x, p) = p.y = M$, then MUP for the same p and M will be solved by the bundle y. This is the duality link. Most of the results of demand theory relate to properties of the compensated demand function:

(i) $f(p, E(p, x)) = f(\lambda p, E(\lambda p, x))$ *for any* $\lambda > 0$: homogeneity of degree zero in prices.

Whenever derivatives exist:

(ii) $\dfrac{\partial E(p,x)}{\partial p_i} = f_i(p,E(p,x));\ \dfrac{\partial^2 E(p,x)}{\partial p_i \partial p_j} = \dfrac{\partial f_i(p,E(p,x))}{\partial p_j}$

And

(iii) *The matrix* $\left(\dfrac{\partial f_i(p,E(p,x))}{\partial p_j}\right)$ is negative semidefinite.

Note that the desired or planned transaction of the individual is thus $z = f(p, p.e) - e$; and given the fact that *e* is fixed, we have, whenever derivatives exist:

$$\frac{\partial z_i}{\partial p_j} = \frac{\partial f_i(p,p.e)}{\partial p_j} = \frac{\partial f_i(p,E(p,f(p,p.e)))}{\partial p_j} - (f_j(p,p.e) - e_j)\frac{\partial f_i(p,M)}{\partial M} \qquad (1)$$

The last step, the *Slutsky Equation*, has been called the *Fundamental Equation of Value Theory*. It may be noted that the results of demand theory concern the matrix of partial derivatives of the compensated demand function, the first set of terms in the above expression. This resolution indicates that the effect of a price change may be broken down into the effect of a price change together with a change in income to compensate the individual for the price change (the substitution effect, the first term on the right hand side) and the effect of a pure change in income (the second term on the right hand side, the income effect); the second set of terms could be of any sign. This is the source of indeterminacy in many areas in microeconomics.

2.1.2 Market Demand Functions

With many individuals or households, indexed by $h = 1, 2, \ldots, N$, each with an utility function $U^h(\cdot)$ defined over the individual's consumption possibility set X^h and an endowment e^h, one may define the demand function for each h as in the previous section, $f^h\left(p, p.e^h\right)$, and the market demand function may now be defined as $X\left(p, \left\{p.e^h\right\}\right) = \sum_h f^h\left(p, p.e^h\right)$. The aggregate demand, or the market demand however cannot, in general, be considered to be derived from some optimization exercise. If, for example, $N = 1, X(p, .)$ coincides with $f^1\left(p, p.e^1\right)$ and hence the properties of the function $X(p, \cdot)$ coincide with the property of the demand function analyzed in the previous section, namely, that it is obtained by solving a problem such as MUP described earlier.

If $E = \sum_h e^h$, the above may be reduced to an enquiry whether the function $X\left(p, \left\{p.e^h\right\}\right)$ may be taken to be $f(p, p.E)$ which is obtained from the maximization of some aggregate welfare subject to an aggregate income constraint. In this connection, the following may be noted:

If for each household h, $U^h(\cdot)$ is homogeneous of degree one, and if endowments e^h are proportional, i.e. $e^h = \delta_h E$, $\forall h$, $\delta_h \geq 0$, *and* $\sum_h \delta_h = 1$ then the market demand $X(p)$ is generated from the following MUP:

$$Max\ U(x) = \prod_h \left(U^h\left(x^h\right)\right)^{\delta_h}$$

$$subject\ to\ p.\sum_h x^h = p.E$$

For future reference, one should note that it is only under some special condition such as mentioned above, that the market demand may satisfy *Weak Axiom of Revealed Preference* (WARP); any demand function derived from a problem of the type MUP, satisfies the following: if $x = f^h\left(p,\, p.e^h\right)$ and $y = B\left(p,\, p\cdot e^h\right)$; then $y = f^h\left(p',\, p'.e^h\right) \Rightarrow x \notin B\left(p',\, p'\cdot e^h\right)$. It is this property which is called WARP; however, the market demand $X(p)$ need not satisfy this rationality property in general, i.e. $X = X(p),\ p.y \le p.E \text{ and } y = X(p')$ need not imply that $p'.X(p) > p'.E$

Even though the market demand function may, in general, fail to satisfy WARP, there are also other results where aggregation is seen to provide helpful regularizing effects. This is seen in terms of the average (per-consumer) demand when there is a continuum of households.

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Biographical Sketch

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